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BOUNDARY LAYER STABILITY AND TRANSITION

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INTRODUCTION

We are indebted to Prandtl (1904) for introducing us to the notion that in flows over solids at high Reynolds numbers ("bei sehr kleiner Reibung"), the effects of viscosity are important only in a thin layer in the neighborhood of the boundary of the solid. That this boundary layer flow was not necessarily laminar but could also be turbulent was pointed out in early experiments by Froude, Eiffel and Prandtl. While for flat plates, the suspected turbulence resulted in larger values of skin friction than in laminar flow, the drag coefficient of a sphere displayed a dramatic decrease beyond a "critical" Reynolds number. Prandtl (1914) successfully explained this latter phenomenon as resulting from the transition of the flow in the sphere's boundary layer from laminar to turbulent ahead of separation. The ability of the turbulent boundary layer to sustain larger adverse pressure gradients than laminar boundary layers moved the separation point downstream increasing the degree of flow attachment and consequently reducing the drag.

It is seen even from these early examples that the understanding and prediction of the flow characteristics of vehicular shapes requires knowledge of transition behavior in addition to the characteristics of laminar and turbulent boundary layers. Nowadays, we have many more such examples over the entire range of aerodynamic speeds, from Mach number zero to the limits of our hypersonic experience.

An early hypothesis on the mechanism of transition from laminar to turbulent flow is that due to Reynolds and developed further by Rayleigh. This hypothesis, that transition is a consequence of instabil-

ity of the laminar boundary layer, remains most highly regarded by workers in the field. It has certainly stimulated much theoretical and experimental work in boundary layer stability. (See Betchov and Criminale 1967 for a fairly up-to-date summary of this work). The excellent agreement between the boundary layer stability experiments of Schubauer and Skramstad (1943), Liepmann (1943), Laufer and Vrebalovich (1960) and Kendall (1967) with appropriate theories provides a basis for proceeding in developing the consequences of the Reynolds-Rayleigh hypothesis.

Nevertheless, transition data have been accumulated and correlated over the years quite independently of stability considerations. These efforts have yielded neither a transition theory nor any even moderately reliable means of predicting transition Reynolds numbers.

In the last two to three years (Morkovin 1968, Reshotko 1968, Mack 1968, Morkovin 1969) attention has again focused on the importance in the transition process of the response of the boundary layer to the available disturbance environment. A significant start toward incorporating such considerations into transition experiments has been reported very recently by Wagner et al. (1969).

One may view the transition of the boundary layer to a turbulent state as the nonlinear response of a very complicated oscillator - the laminar boundary layer - to a random forcing function whose spectrum is assumed to be of infinitesimal amplitude compared to the appropriate laminar flow quantities. The initial response to this random disturbance is covered by infinitesimal disturbance considerations on which there is now a considerable theoretical literature as well as a small but significant experimental literature.

Some remarks are in order about the infinitesimal disturbance theory in which the response of the boundary layer is described by linearized equations. An infinitesimal disturbance is one where the amplitude is insufficient to alter the basic flow whose stability is being studied.* Disturbances are referred to as large or finite when

*The appropriate parameter is a^2Re where a is the dimensionless amplitude and Re is a thickness Reynolds number. For infinitesimal disturbances $a^2\text{Re} << 1$.

they become of sufficient amplitude for the time-independent or time-averaged flow quantities to depart from their laminar values.

It would seem most desirable to formulate stability theory in a way that simulates experiment-namely, to take a given initial disturbance spectrum and follow it forward in time. The response would of course depend on the particular disturbance spectrum assumed and so this treatment is akin to the numerical experiments that are becoming increasingly popular these days. However, in the linear limit of infinitesimal disturbances, the initial disturbance spectrum may be composed in a Fourier sense from a complete set of orthogonal normal modes. The nature of each of these normal modes is determined from the solution of the eigenvalue problem arising from the consideration of the linearized disturbance equations subject to appropriate boundary conditions. Boundary layer stability analyses generally utilize the normal modes approach. The normal modes representation of a disturbance spectrum however does not extend conveniently to finite amplitude and so the nonlinear processes between initial instability and the completion of the transition process are to date hardly understood.

Thus, the relationship between transition Reynolds number and some representative Reynolds number from infinitesimal disturbance stability theory, is quantitatively nebulous and only moderately strong qualitatively. Conversely when it comes to evaluating experimental transition data, the results of stability theory can only serve as a guide.

In this paper the normal modes procedures as they apply to boundary layers will be briefly reviewed and the mechanism of instability discussed. It will then be indicated how normal modes results may be used to give guidance regarding the factors affecting transition. Finally some remarks will be made about the prediction of transition and about the fixing of transition.

NORMAL MODES PROCEDURES FOR BOUNDARY LAYERS

The normal modes methods can be generally described as follows: Let each flow quantity be composed of its value for the specified basic flow plus a disturbance component

$$Q = \overline{Q}\ (\vec{x},\ t) + Q'\ (\vec{x},\ t) \qquad (1)$$

For most problems $\overline{Q}$ is independent of time. The time variation is left in here momentarily in deference to those who study the stability of basic flow patterns that are time dependent (e.g. Shen, 1961, Yang and Kelleher 1964).

The total flow satisfies the time dependent conservation laws of mass, momentum and energy, while the basic flow satisfies a more restricted set of equations. If one is studying the stability of steady laminar boundary layers, then the basic flow equations are the steady boundary layer equations. Subtraction of the basic flow equations from the total flow equations yields the set of conservation law equations satisfied by the disturbances. Since it is stipulated that the fluctuation amplitudes are very small compared to the basic flow quantities, products and squares of fluctuation quantities are neglected. The resulting equations are then linear partial differential equations in the variables $(\vec{x},\ t)$.

Parallel Flow Assumption

The equations can be further simplified by treating the boundary layer as a parallel flow. By a parallel flow, we mean one whose streamlines are everywhere parallel to each other and parallel to any bounding surface.* Strictly speaking, growing boundary layers are not parallel flows. It has however been shown (Cheng 1953, Dunn 1953) that to leading asymptotic approximation, the parallel flow approximation is valid for boundary layers. It has been customary to treat the boundary layer as a parallel-flow even to higher approximation

*This definition is for a two dimensional flow. For a three dimensional parallel flow, the streamlines all lie in parallel planes which are also parallel to any bounding planes.

as evidenced by the extensive numerical solutions of Mack (1965). Brown (1967) has included some non-parallel effects in his calculations but in such a way that their importance cannot be precisely ascertained.

Form of Disturbance

Under the parallel flow assumptions, terms involving the mean normal velocity and longitudinal derivatives of mean quantities are omitted. The stability of a local flow is calculated as if its profiles existed from $-\infty$ to $+\infty$ in a boundary layer of constant thickness. Under these circumstances the profiles are functions of the normal coordinate y only and the equations admit of a disturbance of the form

$$Q'(\vec{x}, t) = q(y)\, e^{i\alpha(x \cos\psi + z \sin\psi - ct)} \tag{2}$$

This is the equation of an oblique plane wave propagating at angle ψ with respect to the x direction. The wave number of the disturbance is α ($\alpha = \frac{2\pi}{\lambda}$ where λ is the wavelength) and c is the phase velocity of the disturbance. The disturbance may be assumed to grow spatially (S: α complex and αc real) or temporally (T: α real and c complex). Disturbances which neither grow nor decay are referred to as neutral. Despite the obvious compatibility of the spatial description to the growth of disturbances in boundary layers, the temporal description was used almost exclusively until about 1964 when Kaplan (1964) presented results based on his method of exact numerical integration of the Orr-Sommerfeld equation. Of course most interest until that time was in defining the boundaries of neutral stability in which limit the two alternatives degenerate to the same analytical problem. For disturbances propagating in the flow direction of a two-dimensional boundary layer Gaster (1962) has shown that in the limit of small amplification the spatial and temporal descriptions both yield c_r as the phase velocity and that the growth rates in the two descriptions are related through the group velocity as follows:

$$\alpha_i\,(S) \approx -\frac{\alpha c_i\,(T)}{c_g} \tag{3}$$

where the group velocity

$$c_g = \frac{\partial(\alpha c_r)}{\partial\alpha} = c_r + \alpha \frac{\partial c_r}{\partial\alpha} \tag{4}$$

In most boundary layer problems, the amplification rates are sufficiently small for Gaster's relation to hold. For more general situations such as two-dimensional flows subject to oblique plane wave disturbances and for three-dimensional flows to arbitrary plane wave disturbances, the relationship is not quite as simple. This is because the phase and group velocities are vector quantities but are not necessarily in the same direction.

In summary, the disturbances considered in normal modes analyses are plane waves. For a two-dimensional boundary layer those disturbances that propagate in the direction of boundary layer development (local free-stream direction) are called two-dimensional disturbances, while those propagating at some angle to the local free stream direction are called three-dimensional disturbances. For a three-dimensional boundary layer where the local free stream is not in the direction of the pressure gradient and, therefore, one might say that there is no single direction of boundary layer development, it is convenient to consider all disturbances as three-dimensional and to identify them by the angle ψ of the direction in which they propagate relative to the reference, say x, direction.

For boundary layers in incompressible and subsonic flow, the phase velocities of the normal modes are generally within the velocity spread of the basic flow. For boundary layers in supersonic flow, one generally deals only with "subsonic" disturbances that is disturbances that move subsonically with respect to the component of the free stream in the direction of wave propagation. Such disturbances have amplitudes that decay exponentially in the free stream. A disturbance that propagates supersonically with respect to the free stream would be expected to have a nonvanishing amplitude far from the wall.

A most important development in recent years in the stability of supersonic laminar boundary layers is the discovery of the higher modes.

Following the questioning of the uniqueness of subsonic disturbances, (Lees and Reshotko 1962), Mack (1965) encountered the higher modes in his numerical calculations and Lees (Lees and Gold 1966) confirmed the conditions for the existence of these additional modes. We now know that if the wall is supersonic relative to the phase velocity of infinitesimal disturbances ($\frac{c_r}{a_w} > 1$), then the boundary layer is rich in unstable normal modes, some of which are not damped by cooling (Mack, 1965). For insulated surfaces, higher modes appear for $M > 2.2$; however it is not until the Mach number is of the order of four or greater that the second mode is at low enough frequency to have experimental consequences. With cooled walls, since for subsonic disturbances $c_r > 1 - \frac{1}{M}$. the higher modes can be significant at Mach numbers as low as 1 if the cooling is sufficient.

Properties of Disturbance Equations

The disturbance equations derived by the procedures indicated in this section* have been shown to display the following properties:

With regard to the stability of two-dimensional parallel flows to three-dimensional disturbances, Squire (1933) has shown that for an incompressible fluid, the disturbance equations can be transformed to the completely two-dimensional Orr-Sommerfeld equation and that the two-dimensional disturbance is the least stable. Dunn and Lin (1955) considered the stability of a two-dimensional compressible boundary layer to three-dimensional disturbances. They showed that when only the leading viscous-conductive effects on the disturbances are considered the equations for three-dimensional disturbances can be transformed to those for two-dimensional disturbances. They carefully pointed out that for compressible flow these transformed equations are not the equations of a proper two-dimensional disturbance so that no "families of solutions" are obtainable; however, the transformation does permit the use of solution procedures for two-dimensional disturbances in problems of three-dimensional disturbances.

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* The complete disturbance equations for a three-dimensional compressible parallel flow subject to an arbitrary plane wave disturbance are derived and stated in Reshotko (1962).

The stability of three-dimensional boundary layers to three-dimensional disturbances is considered for incompressible flow by Owen and Randall (1953) and by Gregory, Stuart, and Walker (1955). Their results for a parallel flow have been concisely summarized by Moore (1956): "For a disturbance assumed to be moving in a certain direction, the eigenvalue problem may be treated as a two-dimensional one, governed by the boundary-layer velocity profile measured in that direction." Of course, for incompressible flow the energy equation is irrelevant and within the framework of the parallel flow assumption this statement is exact. It is shown for compressible flows (Reshotko, 1962) that the transformation implied by Moore's statement applies exactly for the continuity and momentum equations but only for the leading terms of the energy equation. As already pointed out by Dunn and Lin (1955) the dissipation terms do not all transform. Mack (1967) has recently compared results for first mode disturbances with (eighth-order system) and without (sixth-order system) the non-transforming terms and has found the differences in amplification rate to be generally less than 10%. The differences are most pronounced at low Reynolds numbers as would be expected.

Results of Normal Modes Calculations

The results of normal modes calculations are usually presented in diagrams of wave number α versus a thickness Reynolds number. Such diagrams for three different Mach numbers are shown in Figure 1. Since the dimensionless frequency $\frac{\omega\nu}{U^2}$ can be written

$$\frac{\omega\nu}{U^2} = \left(\frac{\alpha}{Re}\right) c_r$$

and since for a given frequency c_r varies very little, a line of constant frequency is almost a straight line through the origin of the α-Re diagram. Note that when higher modes are present, a given frequency may correspond to progressively higher modes as the Reynolds number is increased, or else may excite the higher modes without exciting the first mode. The Reynolds number below which all wave numbers are damped is termed the minimum critical Reynolds number.

Stability diagrams of the results available to 1966 may be found in Betchov and Criminale (1967). The reported results of Mack's extensive calculations for flat plate boundary layers are to be found in Mack (1969).

A rather concise summary of boundary layer stability characteristics as presently understood is given through the following figures taken from Mack (1969).

Figures 2 - 4 describe the characteristics of the most unstable first and second mode frequencies for insulated boundary layers. The data are for Re = 1500 which corresponds to a length Reynolds number of 2.25×10^6. This is enough ahead of observed transition Reynolds numbers so that the stability results are relevant. In figure 2, the dimensionless frequencies are shown. To be noted is that the most unstable first mode frequencies at supersonic speeds occur for oblique waves with ψ generally between 45^o and 65^o while the most unstable second (and higher) mode frequencies occur for $\psi = 0^o$. The $M_1 = 0$ point is of course for $\psi = 0^o$ by virtue of Squire's theorem. The associated temporal and spatial growth rates are shown in figures 3 and 4 respectively. The second mode once activated, clearly displays higher growth rates than the first. To be noted also is the rapidity in decline of spatial growth rate with Mach number particularly around Mach number zero.

The effect of surface cooling on stability is of significance because of the great variety of aerodynamic applications which require cooling. The first mode is generally stabilized by cooling. In fact two-dimensional first mode disturbances can be completely stabilized by cooling up to Mach numbers of the order of 9 (Lees 1947, Dunn and Lin 1955, Reshotko 1963). While the oblique waves cannot all be completely stabilized, it is expected that cooling greatly increases minimum critical Reynolds numbers and diminishes growth rates. On the other hand, the higher modes are not stabilized by cooling. They tend toward higher frequency and higher growth rate as the surface temperature is reduced. Stability diagrams for two-dimensional disturbances at $M_1 = 5.8$ with different degrees of cooling are shown in figure 5.

It is seen that when the temperature level has decreased to $\frac{T_w}{T_r} = 0.25$, the first mode has completely disappeared while the second mode bulge has shifted to higher wave numbers (higher frequencies). The effect of surface temperature on growth rate at $M_1 = 5.8$ is shown in figure 6 in the inviscid limit. The effect of cooling on growth rate in the inviscid limit of the first four modes at $M_1 = 8$ and $M_1 = 10$ are shown in figures 7 and 8 respectively. The upward shift with cooling in both frequency and growth rate is apparent.

It is curious that for boundary layers in water, the effect of cooling is destabilizing while the effect of heating is stabilizing (Wazzan, Okamura and Smith 1967). Because the viscosity of water decreases sharply with increase in temperature, heating yields a fuller velocity profile while cooling tends to give an inflected velocity profile.

Mechanism of Instability

The early work in hydrodynamic stability and in particular the work of Rayleigh emphasized inviscid aspects of the problem under the generally accepted premise that the effects of viscosity on the disturbance flow could only be dissipative. It was concluded at that time that only inflected profiles were unstable. It remained for the originator of the boundary layer concept, Prandtl (1921), to clearly demonstrate the mechanism by which viscosity could be destabilizing and to show therefore that even non-inflected profiles could be unstable.

If one were to construct a disturbance energy equation, then for a neutral subsonic disturbance, the energy "production" through Reynolds' stress would just equal the viscous dissipation.* The Reynolds' stress $-\rho\,\overline{u'v'}$ is zero for a neutral inviscid disturbance since u' and v' are 90° out of phase. The effect of the viscosity near the wall as explained by Prandtl (1921) is to shift the phase resulting in a correlation between u' and v' and thus yield a Reynolds stress. This Reynolds stress level is cancelled by an equal and opposite drop at the critical layer.

*An early detailed derivation is by Schlichting (1935). For compressible flows, this matter has just been examined carefully by Mack (1969).

Prandtl's argument was redeveloped by Lin (1954, 1955) for an incompressible flow and by Lees and Reshotko (1962) for the compressible boundary layer.

Effect of Surface Curvature

This paper is concerned by and large with propagating waves of the form of equation (2), historically termed Tollmien-Schlichting waves in the boundary layer context. Over concave surfaces Görtler (1940 a, b) showed that the boundary layer is unstable to longitudinal vortex disturbances (Taylor-Görtler vortices) very much akin to the vortices that appear between two cylinders, the inner one rotating and the outer at rest (Taylor 1923). Liepmann (1943) has in fact observed for incompressible flow that transition on convex surfaces occurs at about the same Reynolds numbers as for flat plates while on concave surfaces the transition Reynolds number decreases almost linearly with θ/R from the flat plate result. In this expression, the symbol R denotes the radius of curvature of the plate. A comparison of the calculations of Kaplan (1964) and Smith (1955) shows that for incompressible boundary layers over concave surfaces, the minimum critical Reynolds number for Tollmien-Schlichting instability is lower than that for Taylor-Görtler vortices when $\delta/R < \frac{1}{40,000}$ and vice-versa.

BEHAVIOR SUBSEQUENT TO GROWTH OF INFINITESIMAL DISTURBANCES

It has previously been mentioned that our understanding of the processes from initial instability of the laminar boundary layer to the realization of a fully turbulent boundary layer are qualitatively vague and quantitatively nebulous. This is regardless of speed or even compressibility. We do however expect the time dependent velocity field of a fully turbulent "two-dimensional" boundary layer to be random, nonlinear and three-dimensional. The current notions of how these properties develop will be briefly discussed.

Randomness - The initial disturbance spectrum is generally thought to be random in the sense of absence of discrete peaks in both frequency and orientation. The resulting frequency and orientation spectra of the fully turbulent boundary layer are also expected to be devoid of discrete peaks but they will probably differ greatly from the initial disturbance spectrum. It is generally thought that the spectra of an equilibrium turbulent boundary layer are independent of the developmental history of the boundary layer and that the final spectra after their evolution through nonlinear processes display detailed balance between production and decay at each frequency.

It is not clear that this equilibrium state is generally reached in our experimental turbulent boundary layers particularly at supersonic and hypersonic speeds.

Nonlinearity - The processes leading to transition are fundamentally nonlinear. After initial instability some of the important features of the nonlinear growth are the effects on the frequency and disturbance spectra through distortion of the mean flow, generation of harmonics and beat or resonance phenomena.

Another possible feature of nonlinear processes is the attainment of a metastable equilibrium state at a finite amplitude as suggested by Landau (1944). This question has been examined in some detail by Stuart (1960, 1962) for incompressible plane Poiseuille flow. Because

he could not fully evaluate the relevant terms in his equation for equilibrium amplitude, Stuart could not come to a definite conclusion regarding the existence of a finite amplitude equilibrium state. However, if such a state did exist, then there could be no subcritical instability a la Meksyn and Stuart (1951) and vice versa. This question and that of subcritical instability have yet to be critically examined for boundary layer flow.

Three-Dimensionality - The initial disturbance spectrum is very likely three-dimensional in orientation. Even though not all orientations are amplified at once, certainly a band of them become unstable within the regime of infinitesimal disturbances. Additional orientations might be excited through the "resonance" mechanism suggested by Raetz (1959). Furthermore, spanwise energy transfer and steamwise vorticity can result from the interaction of two-dimensional and oblique waves as pointed out experimentally by Klebanoff, Tidstrom and Sargent(1962) and theoretically by Benney and Lin (1960). These factors all contribute to the three-dimensionality of the eventual turbulent velocity field. To be noted is that the referenced studies are all for incompressible flow.

It is evident that very little is known about finite amplitude behavior for boundary layers and that none of what is known has been developed for compressible boundary layers. Nevertheless, it is felt that the arguments on randomness, nonlinearity and three-dimensionality as developed in the context of low speed flows are in their general sense equally applicable to the compressible, even hypersonic boundary layer.

FACTORS AFFECTING TRANSITION

Whether one proceeds from the discussion of the prior sections or else goes through similitude arguments (Reshotko 1968), it is abundantly clear that in addition to being a function of the mean flow conditions, transition must in some way be related to the wave-number and orientation spectra of the disturbance environment. This was pointed out by Laufer (1954) many years ago and again emphasized by Morkovin (1968) as evidenced by figure 9 taken from his work. The disturbances are identified unequivocally at the top of the diagram as INPUT. The traditional "factors affecting transition" are identified in the diagram as operation modifiers - factors modifying the amplification characteristics of the oscillator. This diagram is well worth studying in that it summarizes in a very concise way the behavior in the linear and early non-linear regimes of instability that may eventually lead to transition.

Based on stability considerations, Reshotko (1968) has deduced the following plausible forms for the relation between the transition Reynolds number and the characteristic dimensionless frequencies and/or wavelengths of the disturbance spectra:

$$(Re)_{tr} \sim \left(\frac{U^2}{\omega\nu}\right)^n \tag{5}$$

$$(Re)_{tr} \sim \left(\frac{U\lambda}{\nu c_r}\right)^n \tag{6*}$$

Equation (5) indicates that for a given disturbance frequency spectrum characterized by ω , the transition Reynolds number will vary with $\frac{U^2}{\nu}$. The coefficient and exponent will be functions of Mach number and surface temperature level and possibly also $\frac{\omega\nu}{U^2}$ to allow for deviations from the power law of Equation (5). Equivalently, equation (6) indicates that for a given wavelength spectrum character-

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*Note that $\frac{\omega\nu}{U^2} = 2\pi\left(\frac{c_r\nu}{U\lambda}\right)$

ized by λ, the transition Reynolds number will vary with U/ν where again the coefficient and exponent will depend on Mach number, surface temperature level and $\frac{U\lambda}{\nu c_r}$. The dimensionless phase velocity is a very slowly varying quantity particularly at hypersonic Mach numbers.

In less abstract language, we are saying that the importance of a given physical (dimensional) frequency or wavelength depends on the amplification associated with that frequency or wavelength. But the amplification depends on the dimensionless frequency or dimensionless wavelength and so the importance of a given physical spectrum (characterized by ω or λ) in leading to transition depends on the associated values of $\left(\frac{U^2}{\nu}\right)$ or $\left(\frac{U}{\nu}\right)$ respectively.

The phenomenon just described through equations (5) and (6) may well be what has been traditionally referred to as the unit Reynolds number effect. From the arguments presented, this effect is to be expected in any facility or test where the spectrum of available disturbances is non-white in the bands that have relevance to instability and transition. Accordingly it may be encountered in any facility. If it is a physical frequency spectrum that remains invariant from one test to the next in a given facility then $(Re)_{tr}$ will depend on $\left(\frac{U^2}{\nu}\right)$. On the other hand, if it is a physical wavelength spectrum of disturbances that remains fairly constant over a range of facility operating conditions, then $(Re)_{tr}$ will vary with U/ν. A combination of the two is also possible.

The discussion so far has for simplicity ignored the orientation spectra of disturbances. These can be readily accommodated. It is known that the growth rate of disturbances is orientation dependent and so it is quite possible that the transition Reynolds number would also show some dependence on orientation spectrum. This dependence has yet to be sought experimentally.

A relevant calculation has recently been performed by Mack (1968). He calculated the response of a $M_1 = 4.5$ flat plate boundary layer to the spectrum of far field sound radiated from the side-wall turbulent boundary layer of the JPL 20" supersonic wind tunnel (Laufer 1964).

He assumed that the spectrum is independent of position in the boundary layer, that the intensity and shape of the spectrum are independent of unit Reynolds number and that the disturbance energy is distributed uniformly through all wave angles. The pertinent response functions are shown in figure 10. In this figure, A is the amplitude of the most unstable constant-frequency disturbance at Re = 1500 (Re_x = 2.25 x 10^6) and A_1 is the amplitude at the start of the unstable region. Similar figures are available for other Reynolds numbers. The power spectral density of the input and output spectra at three different thickness Reynolds numbers are shown in figure 11 for a unit Reynolds number of 1 x 10^5 per inch. The quantity n is the dimensionless frequency, L_x is the integral scale of turbulence and U_s is the average convection speed of the sound sources. Similar results have been calculated for other unit Reynolds numbers. The output amplitudes are shown in figure 12 where it is seen that as unit Reynolds number increases a larger thickness Reynolds number is required to attain a given amplitude. If in turn the transition point is identified or correlated with the attainment of a given disturbance amplitude then the transition Reynolds number would increase with unit Reynolds number as is in fact observed experimentally.

It is probable that few of the assumptions underlying the calculation are strictly correct, but it is believed that the essence of the phenomenon, which is the movement with unit Reynolds number of the unstable frequency band with respect to the input spectrum, has been retained. A more definitive calculation of unit Reynolds number effect must await measurements of the variation of input spectrum with unit Reynolds number and wave angle.

Reshotko (1968) points out another consequence of the dimensionless frequency and/or dimensionless wavelength argument, and that is the tendency of facilities or flight altitudes to emphasize particular modes of instability of the supersonic and hypersonic boundary layer. It is shown there that in the ballistic range tests of Sheetz (1965) at Mach number 5, second and higher mode excitation is highly improbable and so the observed transition behavior is dominated by first mode

considerations. Shock tunnel data reported by Stetson and Rushton (1967) at about the same Mach number and surface temperature level but at an order of magnitude lower value of $\frac{U^2}{\nu}$ show a decrease of transition Reynolds number with cooling as might be expected through the involvement of second and higher modes. It is also shown that in data reported by Deem and Murphy (1965) and by Sanator, et al (1965) at Mach number 10 in VKF Tunnel C, second and higher mode excitation is quite likely and tends to explain the insensitivity of transition Reynolds number to surface cooling.

The prospective involvement of higher modes in a given supersonic or hypersonic situation is as follows: The lower the value of $\left(\frac{U^2}{\nu}\right)$ (therefore higher $\frac{\omega\nu}{U^2}$), the greater the importance of the higher modes. It seems that it may be difficult to escape the higher modes in steady flow hypersonic wind tunnels, while on the other hand, they may have little relevance to transition in a ballistic range.

While the physical disturbance frequencies in flight are unknown, the corresponding dimensionless frequencies $\left(\frac{\omega\nu}{U^2}\right)$ are strongly dependent on Mach number and altitude. This is shown in figure 13 for an assumed frequency of 10 kc. The dimensionless frequency changes by about an order of magnitude for each 50,000 feet of altitude. Thus a 100 kc disturbance at 100,000 feet has the same dimensionless frequency as a 10 kc disturbance at 150,000 feet. The dimensionless frequencies corresponding to 10 kc in each of a number of hypersonic facilities are superimposed. Because of the equality of frequencies, the altitudes indicated for each facility are their $\left(\frac{U^2}{\nu}\right)$ altitudes.

If the disturbance spectrum of a given facility is known, then its corr ponding range of dimensionless frequencies would indicate the range of frequency-altitude combinations simulated.

Again, equivalent arguments may be presented in terms of wavelength through the parameter $\left(\frac{U\lambda}{\nu}\right)$. The values of the dimensionless wavelength $\left(\frac{U\lambda}{\nu}\right)$ for a disturbance having a physical wavelength of one inch at various flight altitudes are shown in figure 14. The values for a one inch wavelength disturbance in each of the facilities of figure 13 are superimposed. To be noted in comparing figures 13 and 14 is that

the $\left(\frac{U}{\nu}\right)$ altitude of a given facility is not necessarily equal to the $\left(\frac{U^2}{\nu}\right)$ altitude. For the cited facilities the $\left(\frac{U}{\nu}\right)$ altitude is slightly lower than the $\left(\frac{U^2}{\nu}\right)$ altitude.

The order of magnitude variations of $\frac{U}{\nu}$ and $\frac{U^2}{\nu}$ with each 50,000 feet of altitude indicate that significant attention must be given to the choice of laboratory test conditions in order to closely simulate a particular dimensionless disturbance environment.

PREDICTION OF TRANSITION

As has been mentioned, the objective of the foregoing presentation is to lead to a rational scheme of predicting transition behavior in wind tunnels as well as in flight.

A significant attempt in accomplishing this objective was by Smith and Gamberoni (1956), who for low speed flow tried to correlate transition Reynolds number over plates, wings and bodies with the amplitude ratio of the most unstable frequency from its neutral point to the transition point. Using theoretical values of c_i from the temporally growing calculations of Pretsch (1942) for the Falkner-Skan profiles, together with experimental data on transition Reynolds number, they found that the transition Reynolds number $Re_{x,tr}$ as predicted by assuming an amplification factor of e^9 was seldom in error by more than 20%. Jaffe, Okamura and Smith (1969) updated the Smith-Gamberoni method by using spatial growth rates calculated by exact solution of the Orr-Sommerfeld equation for the locally observed profiles on the various shapes. They found good correlation with estimations based on an amplification factor of e^{10}.

Despite the apparent success of these procedures, they are defective in principle and perhaps also in practice. Smith (Smith and Gamberoni 1956) acknowledges that the boundary layer is "agitated by disturbances impressed upon it by external turbulence, suffface roughness, noise, and vibration", and that "the true flow is similar to a forced vibration". Yet, the disturbance spectrum is in no way involved in his method and accordingly there is no way of introducing a unit Reynolds number effect. A pointed example of the defectiveness of the method is that it cannot explain why for a flat plate, Schubauer and Skramstad (1943) obtain a transition Reynolds number of 2.84×10^6 while Wells (1967) obtains 4.9×10^6. The difference is no doubt due to the reduction in background noise in the Wells (1967) experiment but there is no accommodation of this fact into the Jaffe et al (1969) procedure. This points out the need for a criterion based on amplitude rather than amplification.

Again it becomes clear that the disturbance environment must be considered in the prediction of transition. In wind tunnels, Pate and Schueler (1969) show that the transition Reynolds number on test models can be correlated with parameters related to the sound radiated from the turbulent boundary layers on the tunnel walls. Kendall at $M_1 = 4.5$ observes no transition on a flat plate whose length Reynolds number is 3.3×10^6 when the tunnel wall boundary layer is laminar. In the same tunnel at the same Mach number but with turbulent side wall boundary layers Coles (1954) observed transition at Reynolds numbers of the order 1×10^6. The measurement of the disturbance spectrum (primarily radiated sound) in wind tunnels and the determination of the exact role that this spectrum plays in the transition process will be quite important in assessing wind-tunnel transition data. A significant start in this direction is by Wagner et al. (1969) who measured the spectra of radiated sound in the Langley Mach 20 Helium Tunnel at different unit Reynolds numbers and conclude that the model transition point was strongly coupled to the strength of the sound pressure level.

Whatever the difficulties of transition prediction in wind tunnels where disturbance spectra are readily measurable, the rational prediction of transition Reynolds numbers in free flight borders on the impossible because of the lack of information on the disturbance environment in free-flight. The traditional ways of extrapolating wind-tunnel data to flight conditions fail to account for the differences in disturbance environment. For example, the extrapolation of a wind-tunnel result to flight unit Reynolds numbers tends according to equation (6) to assume the constancy of a characteristic disturbance wavelength. There is no basis for such an assumption. Morkovin (1969) indicates that there is some present effort in determining the distribution, intensity and scales of disturbances at altitudes up to 200,000 feet. He suggests as an interim working hypothesis that the disturbances at high altitudes have characteristics that are no worse than those at 20,000 - 40,000 feet where considerable information has been and is being gathered in connection with commercial airline operation.

It is felt that any rational procedure for the prediction of

transition should follow the processes of figure 9 as far along as is possible and then try to correlate transition with an amplitude level or a level of distortion of the basic flow. The work of Hack (1968) has demonstrated that such calculations are possible provided that there is adequate INPUT information.

FIXING OF TRANSITION

The achievement of earlier transition through artificial tripping of the boundary layer is often desired in order to simulate turbulent boundary layer behavior at full scale Reynolds numbers. The testing literature is replete with descriptions of the response of the boundary layer to varieties of surface roughness - single roughness elements, multiple elements, spherical bodies, distributed roughness, etc. These trips have been developed more or less by trial and error.

Stability considerations offer us both an explanation for the observed behavior due to various kinds of trips as well as suggestions for more effective tripping. The general decay in spatial amplification rate with Mach number (figure 4) is probably responsible for the increasing difficulty with Mach number of tripping. Also the art of tripping has not really had the chance to benefit from the recent documentation of greater first-mode instability to oblique waves than to two-dimensional waves.

It is suggested that tripping devices be designed so as to capitalize on the known instability characteristics of laminar boundary layers. Referring to figure 4, a trip should generate oblique waves of appropriate wavelength to be most effective at Mach numbers up to 4. Note Hama's success with "triangular patch stimulators" at Mach numbers up to about 5 (Hama 1964). Beyond Mach number 4 it seems desirable to excite the second mode for most efficient tripping. Furthermore, the trips need not be mechanical. It is apparent that radiated sound at appropriate frequencies (figure 2) can have a noticeable effectiveness in promoting transition.

CONCLUDING REMARKS

The process of transition from laminar to turbulent flow remains almost as baffling as the turbulence in the flow that follows it. However, significant inroads into the understanding of transition are now possible because we are presently able to do sophisticated theoretical and experimental studies of the stability of laminar boundary layers. Some of the anomalies of the past are now explained and a greater sensitivity has developed to the details of the instability and growth that are at the foundation of transition.

The lack of knowledge of disturbance spectra in wind tunnel and flight situations is salient at this time.

The fact that over one-third of the references cited in this paper are less than five years old indicates the renewed interest in boundary layer stability and transition and provides hope that our understanding of transition will develop more rapidly than in the past.

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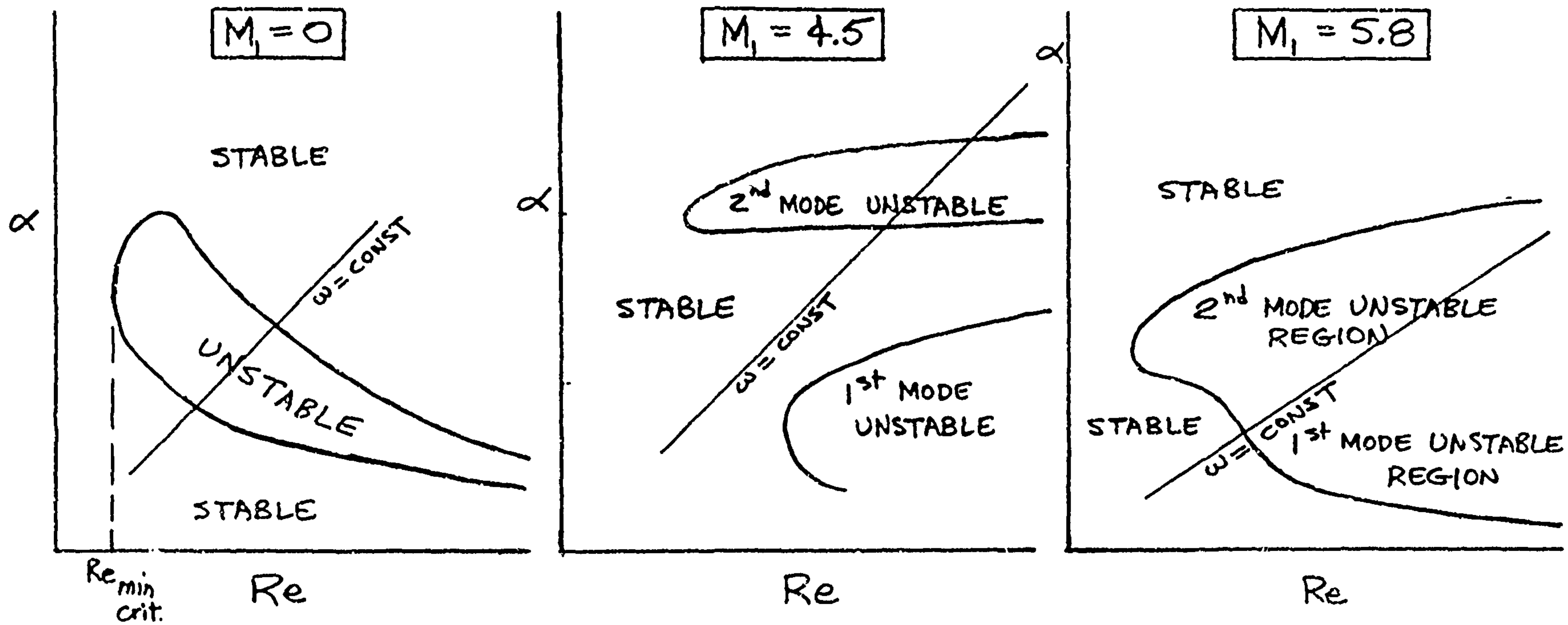

Figure 1. Stability Diagrams for Insulated Boundary-Layers

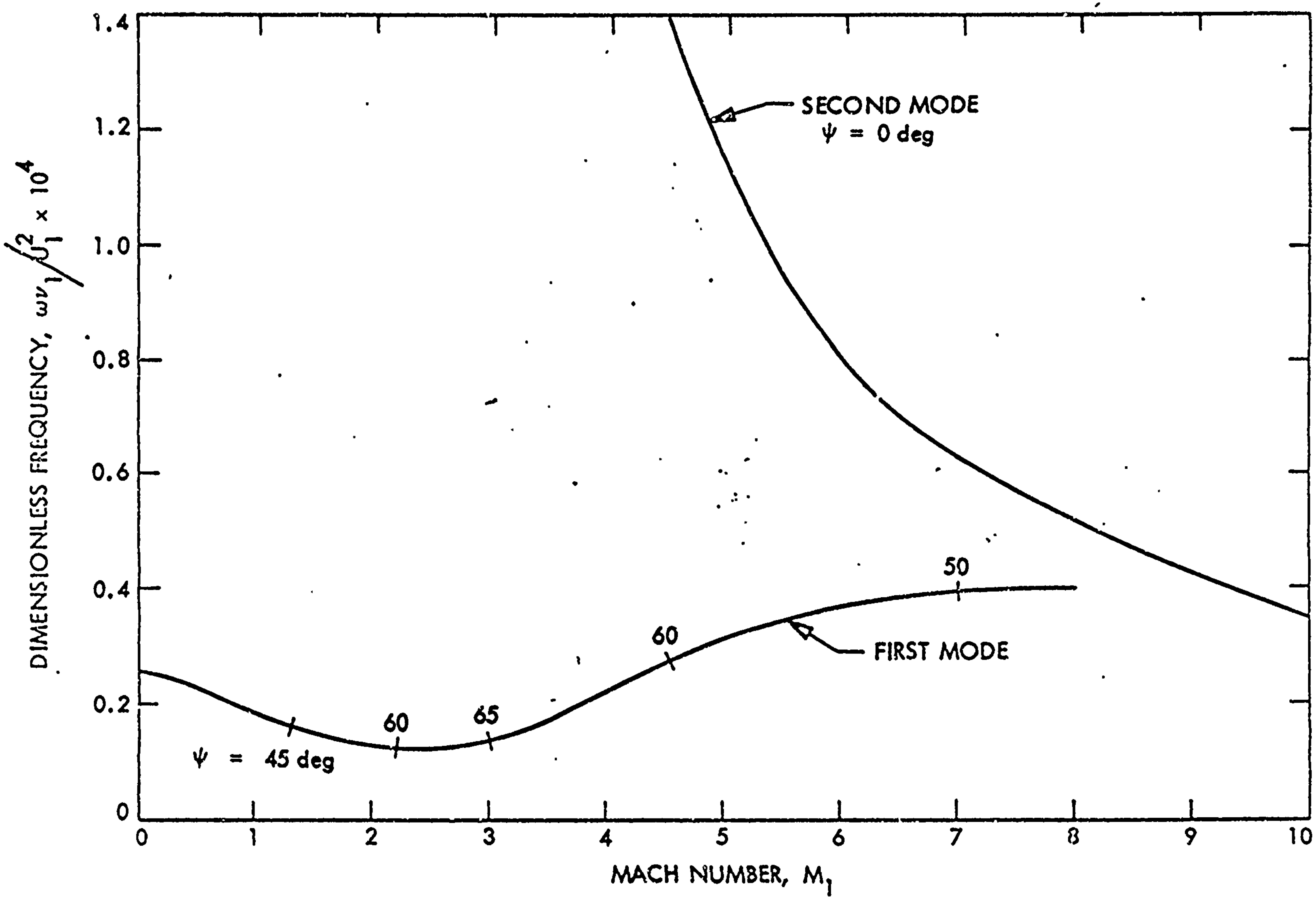

Figure 2. Effect of Mach Number on First and Second Mode Most Unstable Frequencies at Re = 1500. Insulated Wall, Wind-Tunnel Temperatures (Mack 1969)

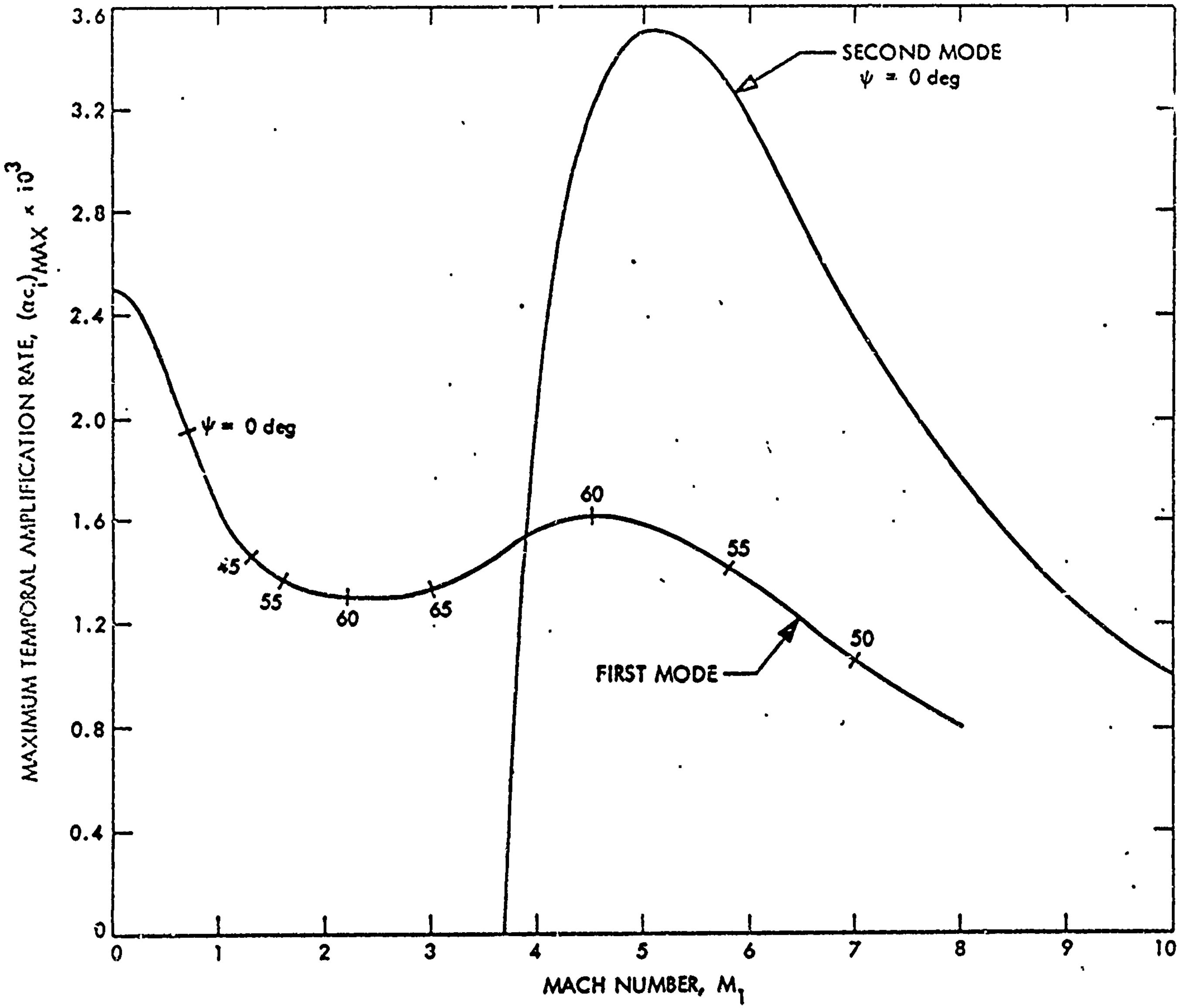

Figure 3. Effect of Mach Number on Temporal Amplification Rate of Most Unstable First and Second Mode Disturbances at Re = 1500. Insulated Wall, Wind-Tunnel Temperatures (Mack 1969)

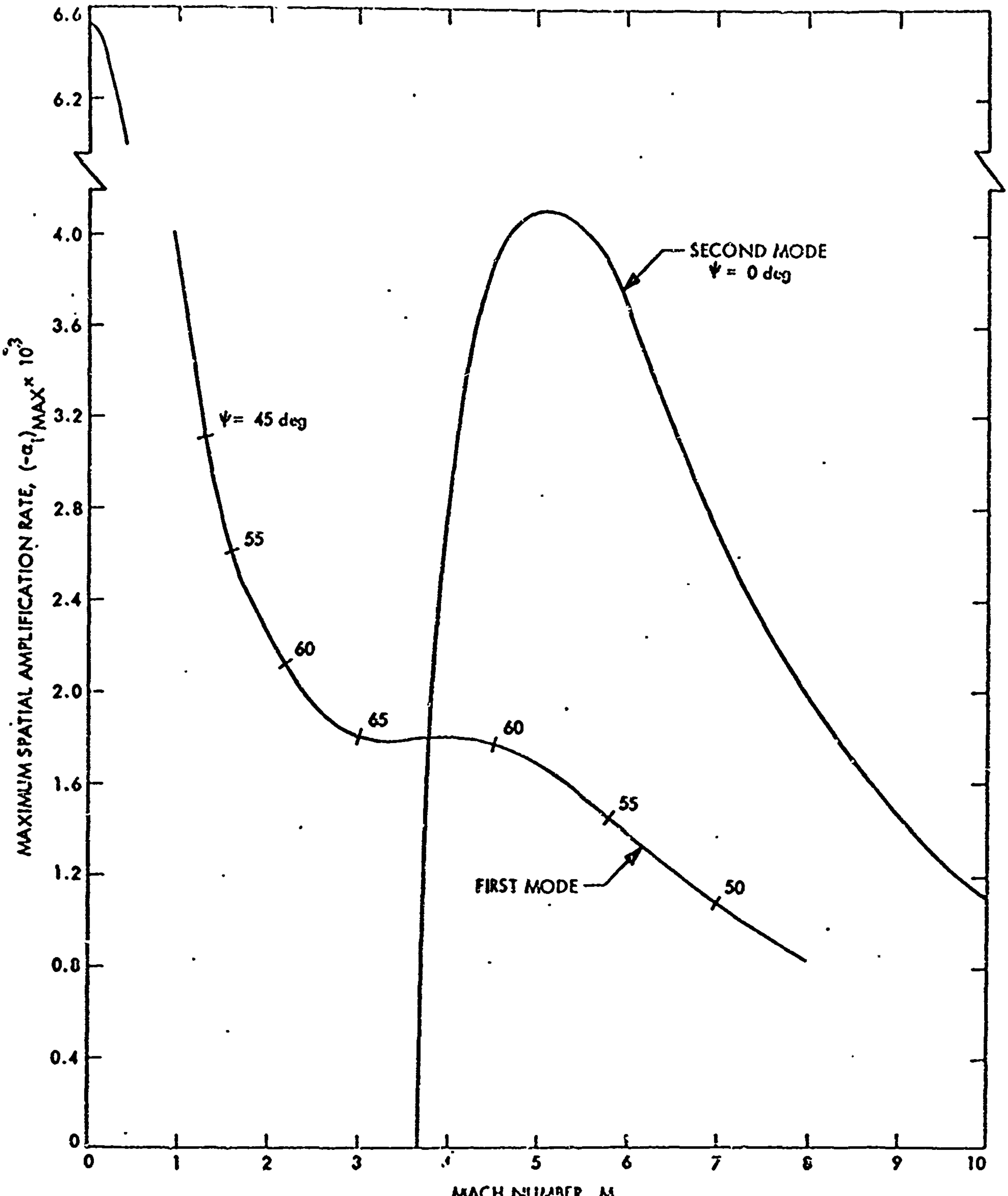

Figure 4. Effect of Mach Number on Spatial Amplification Rate of Most Unstable First and Second Mode Disturbances at Re = 1500, Insulated Wall, Wind-Tunnel Temperatures (Mack 1969)

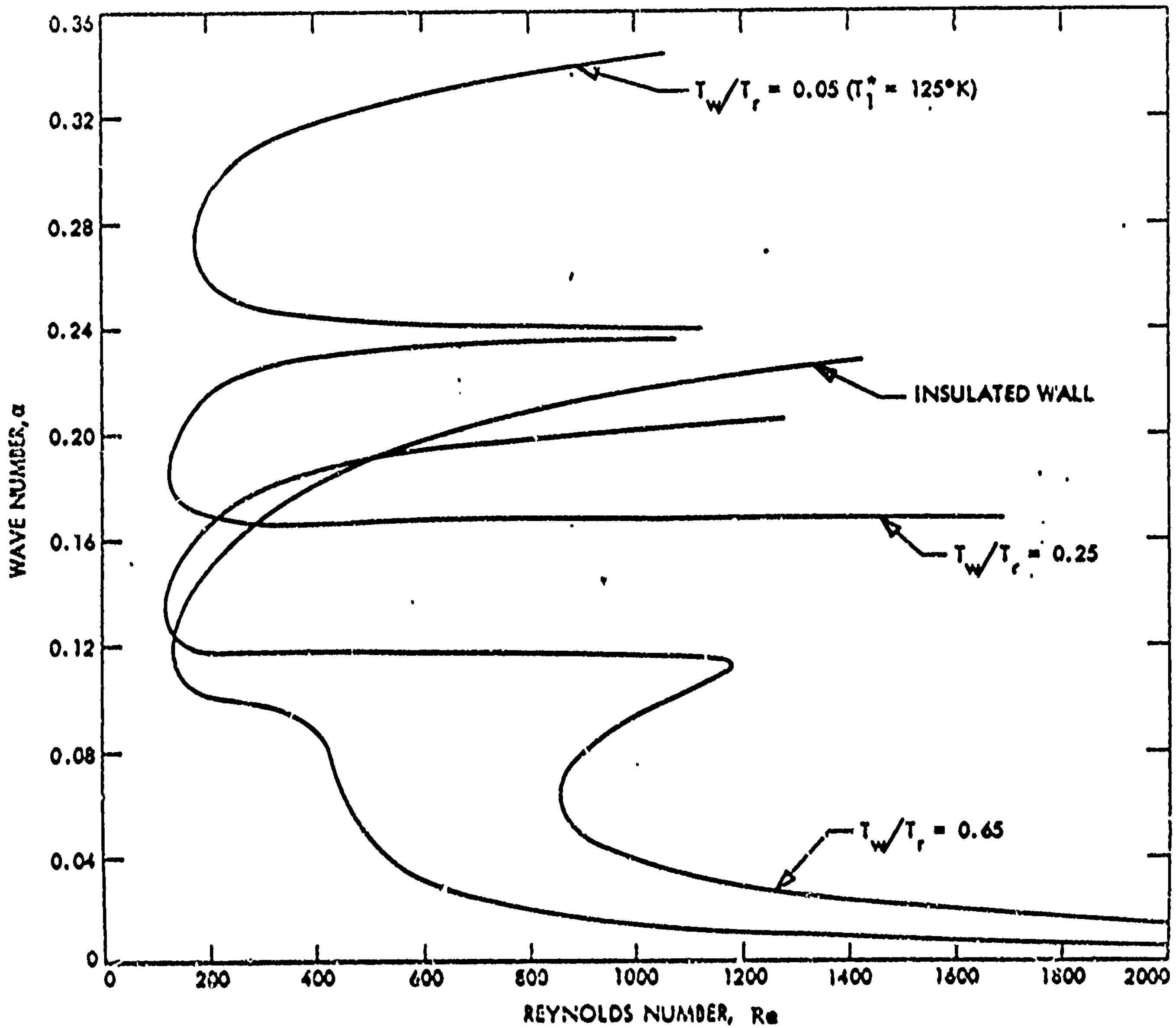

Figure 5. Effect of Wall Cooling on Neutral Stability Curve at M_1 = 5.8. Two-Dimensional Disturbances, T_1^* = 50°K (Mack 1969)

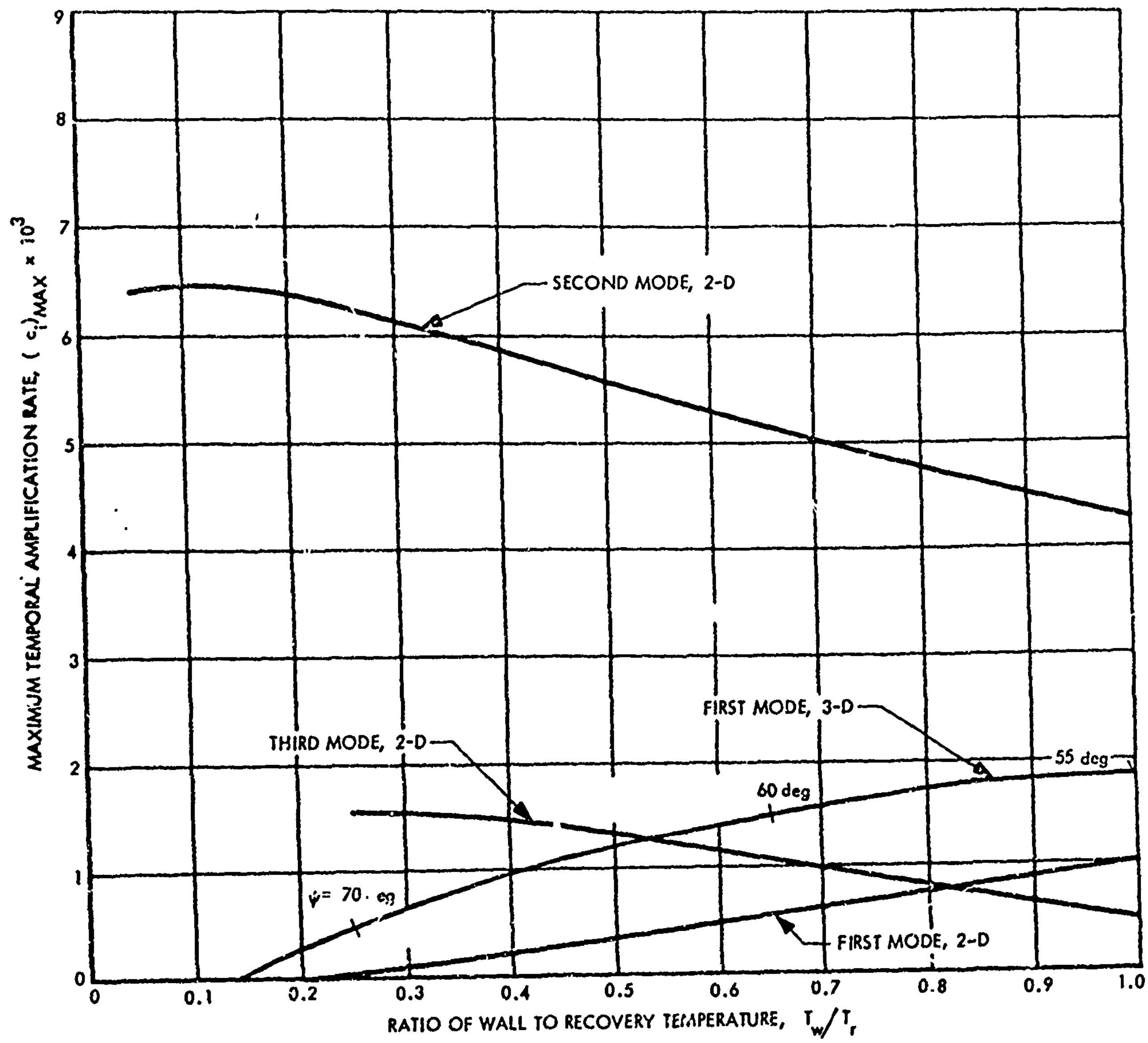

Figure 6. Effect of wall cooling on maximum temporal amplification rate of first three modes at $M_1 = 5.8$. $T_1^* = 50°K$ (Mack 1969)

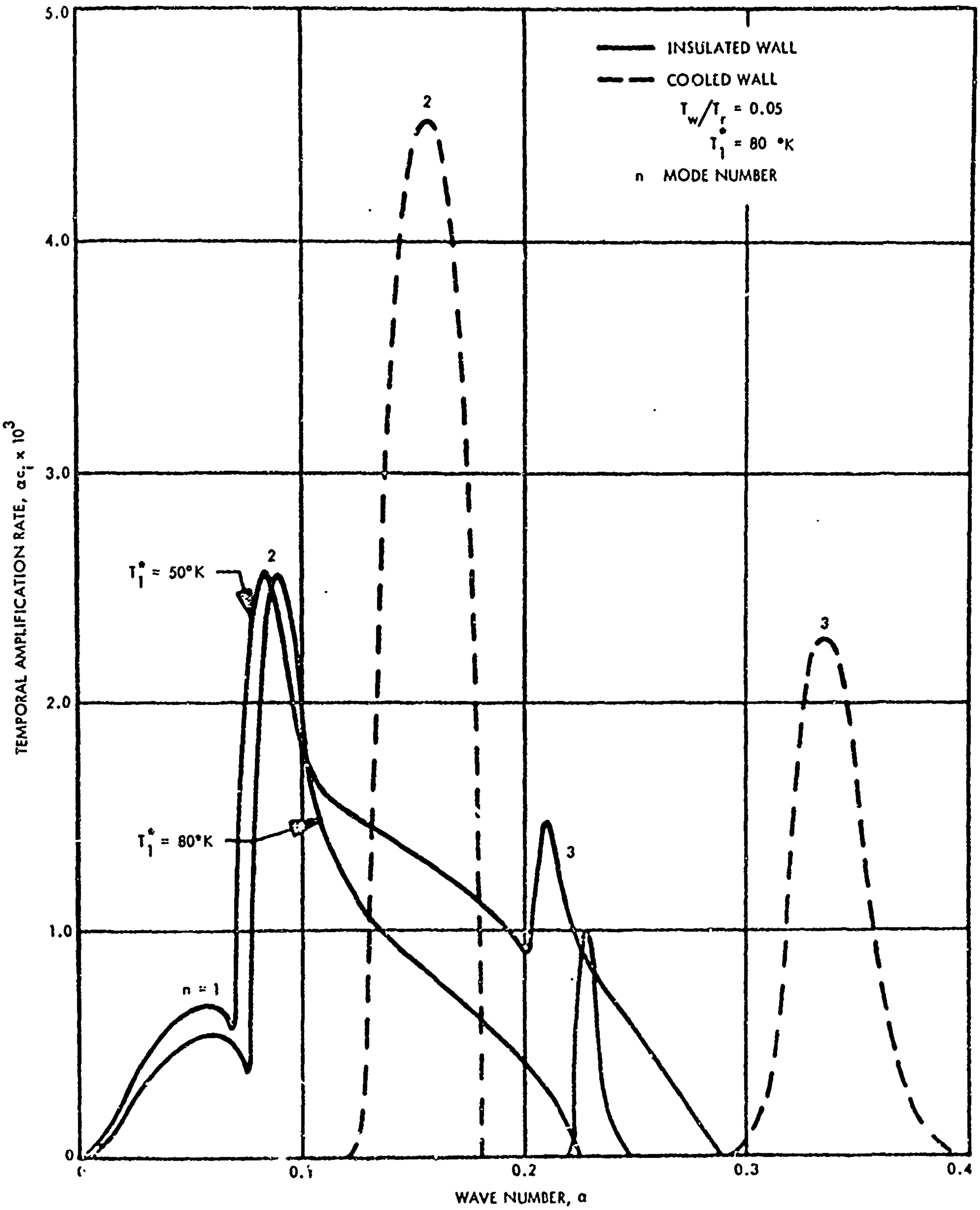

Figure 7. Temporal amplification rate vs wave number for first three modes for insulated wall and cooled wall (M_1 = 8) **(Mack 1969)**

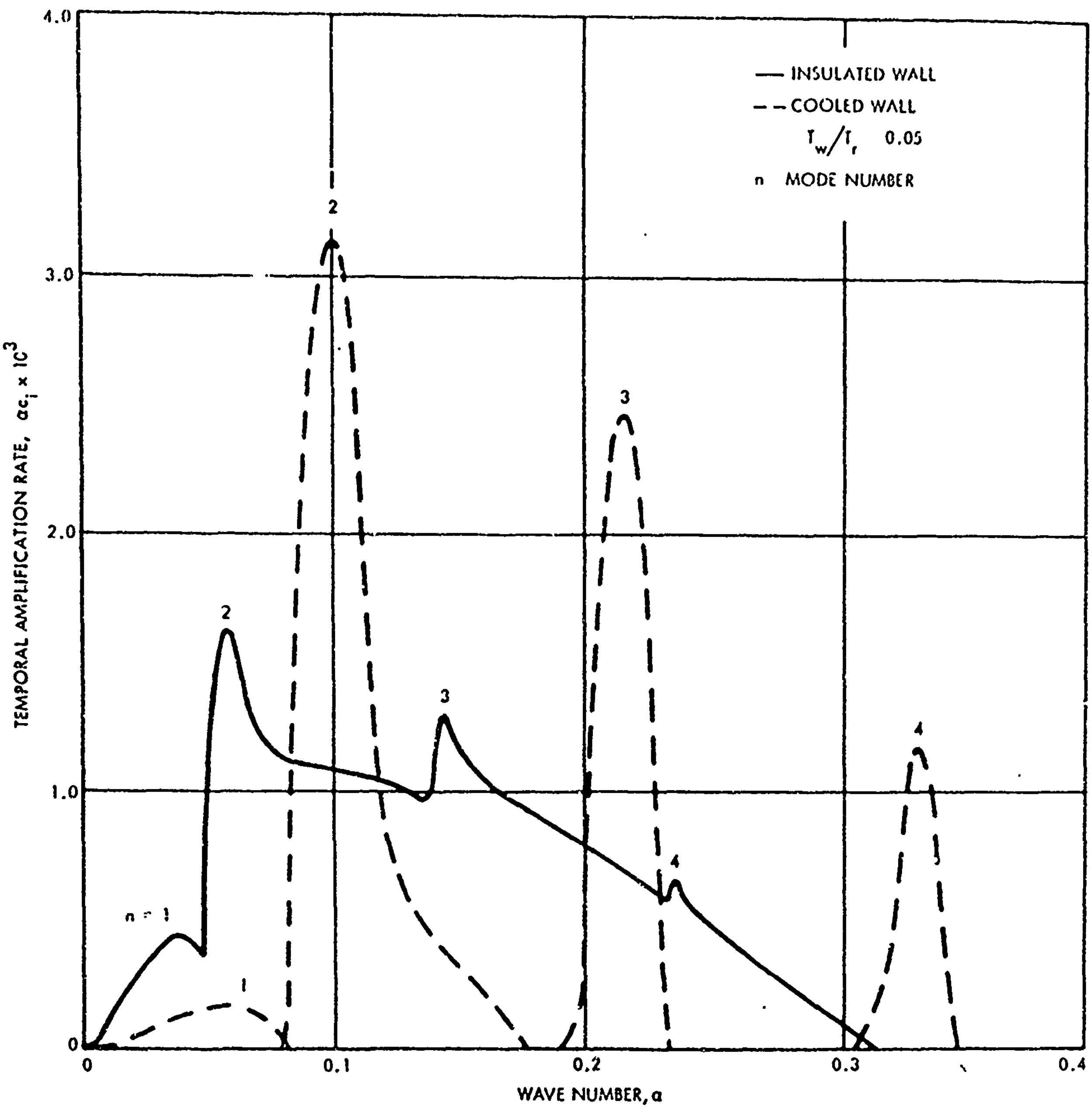

Figure 8. Temporal amplification rate vs wave number for first four modes for insulated wall and cooled wall (M_1 = 10, T_1^* = 50°K) (Mack 1969)

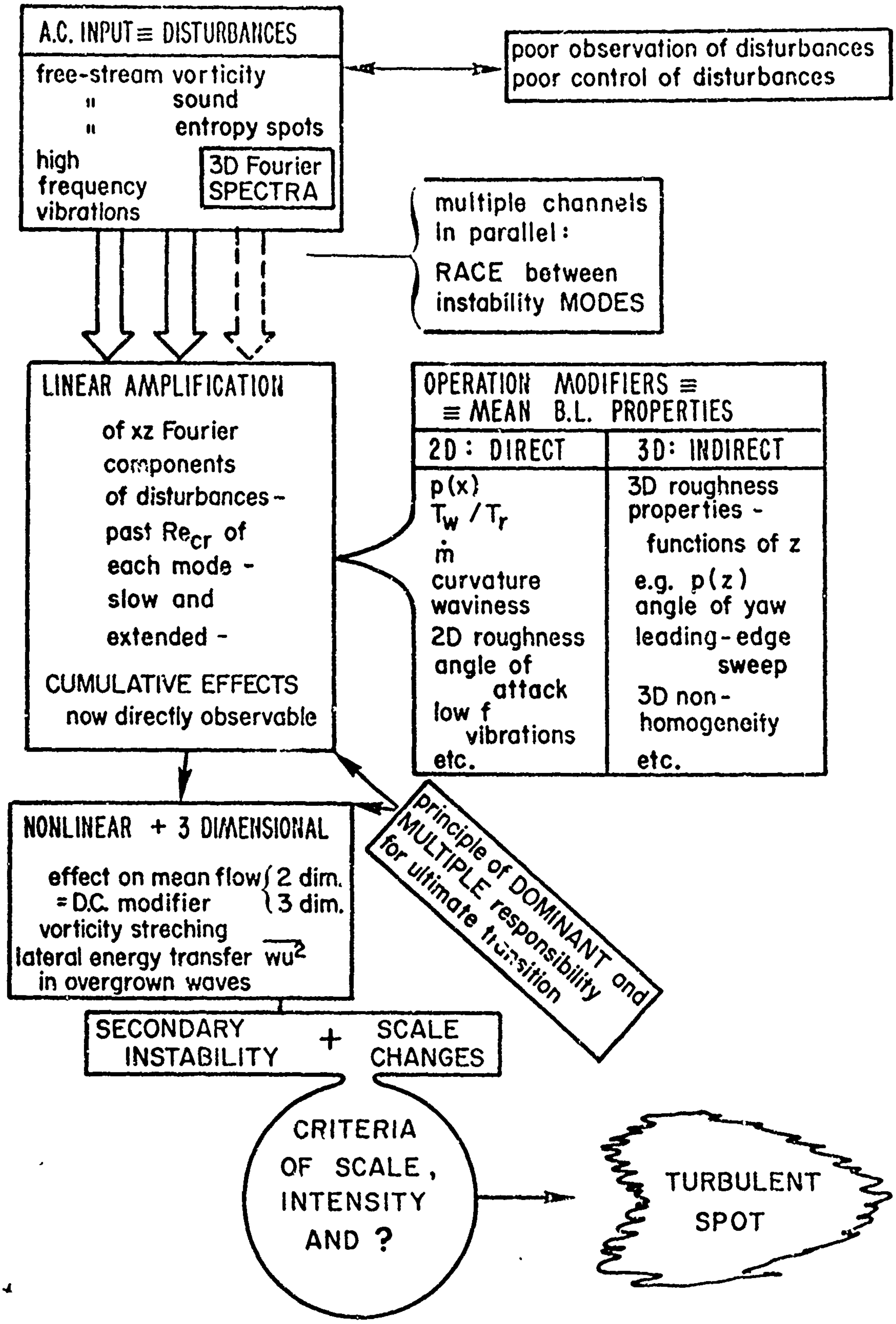

Figure 9. Laminar Boundary Layer as a Linear and Nonlinear Operator (Morkovin 1968)

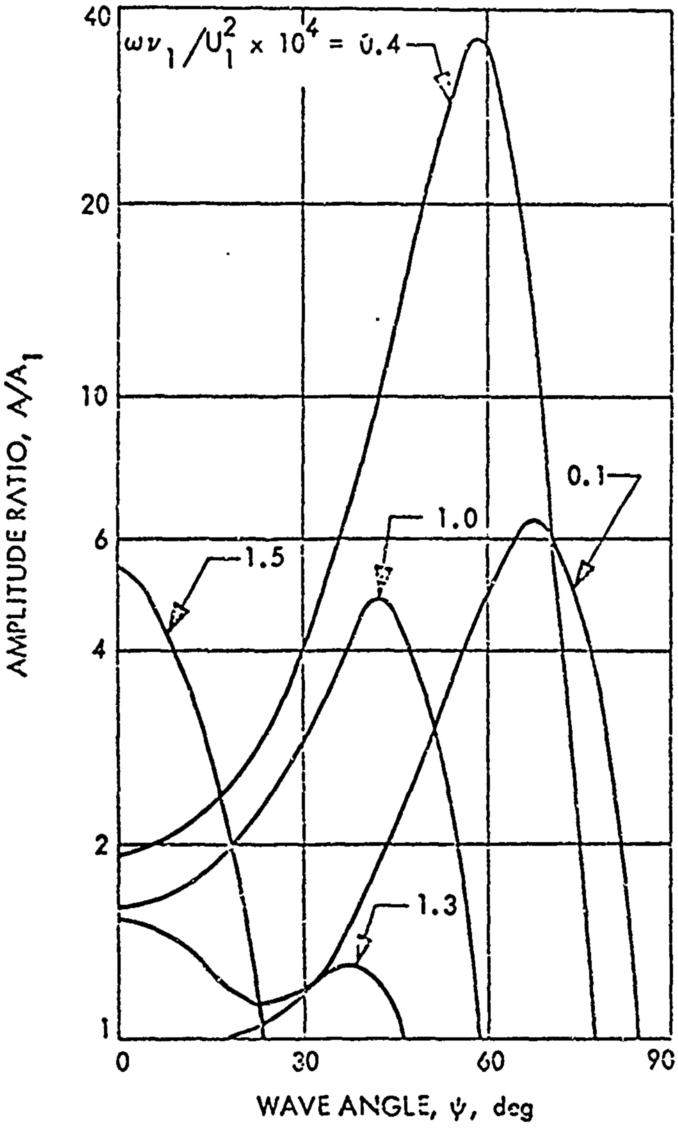

Figure 10. Distribution with Wave Angle of Amplitude Ratio of Constant Frequency Disturbances at Re = 1500. M_1 = 4.5, Insulated Wall, T_0^* = 311°K (Approximate Calculation) (Mack 1969)

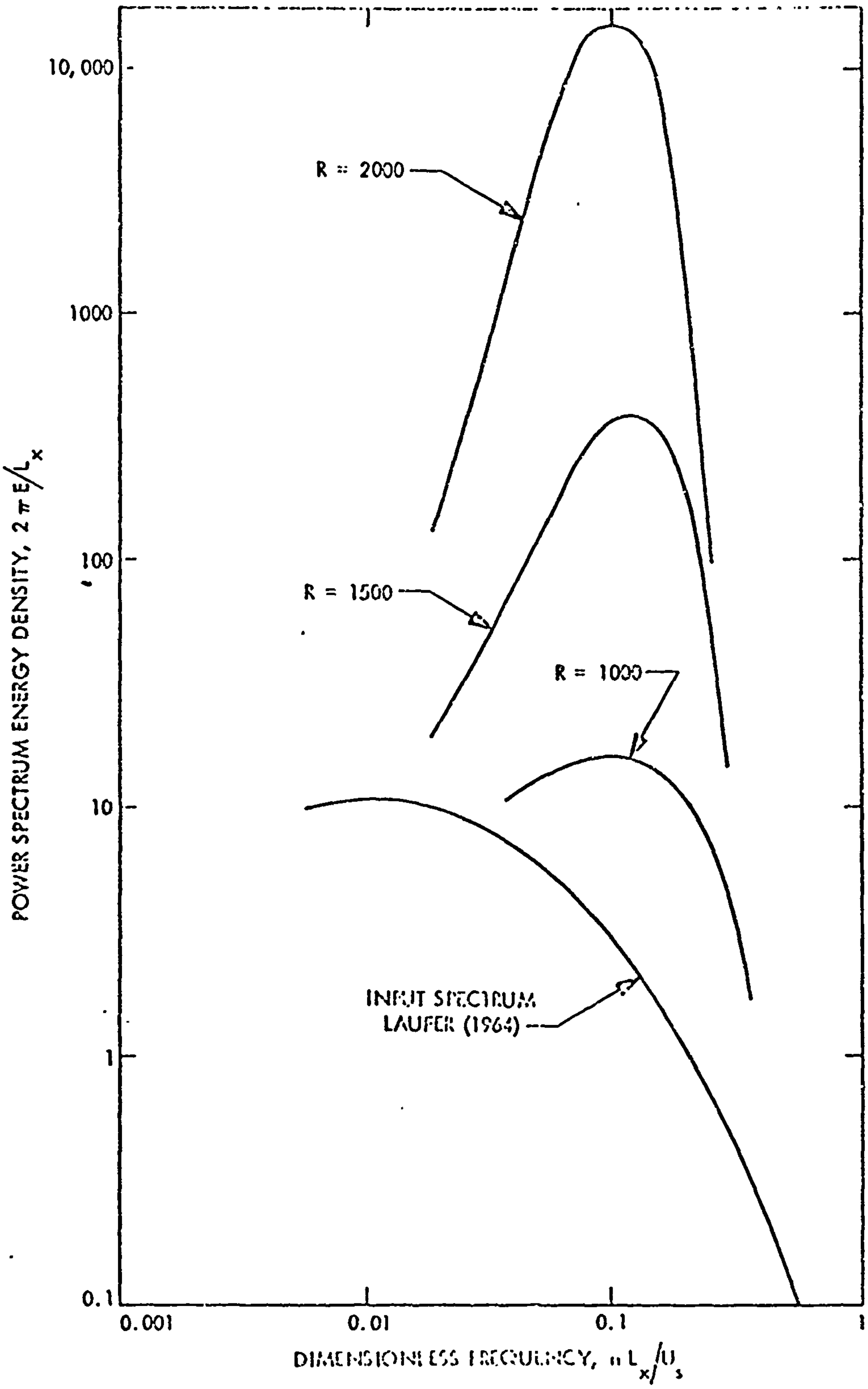

Figure 11. Input and Output Disturbance Spectra, $M_1 = 4.5$, Re/in. = 1×10^5, Insulated Wall, $T_0^* = 311^\circ K$ (Approximate Calculation) (Mack 1969)

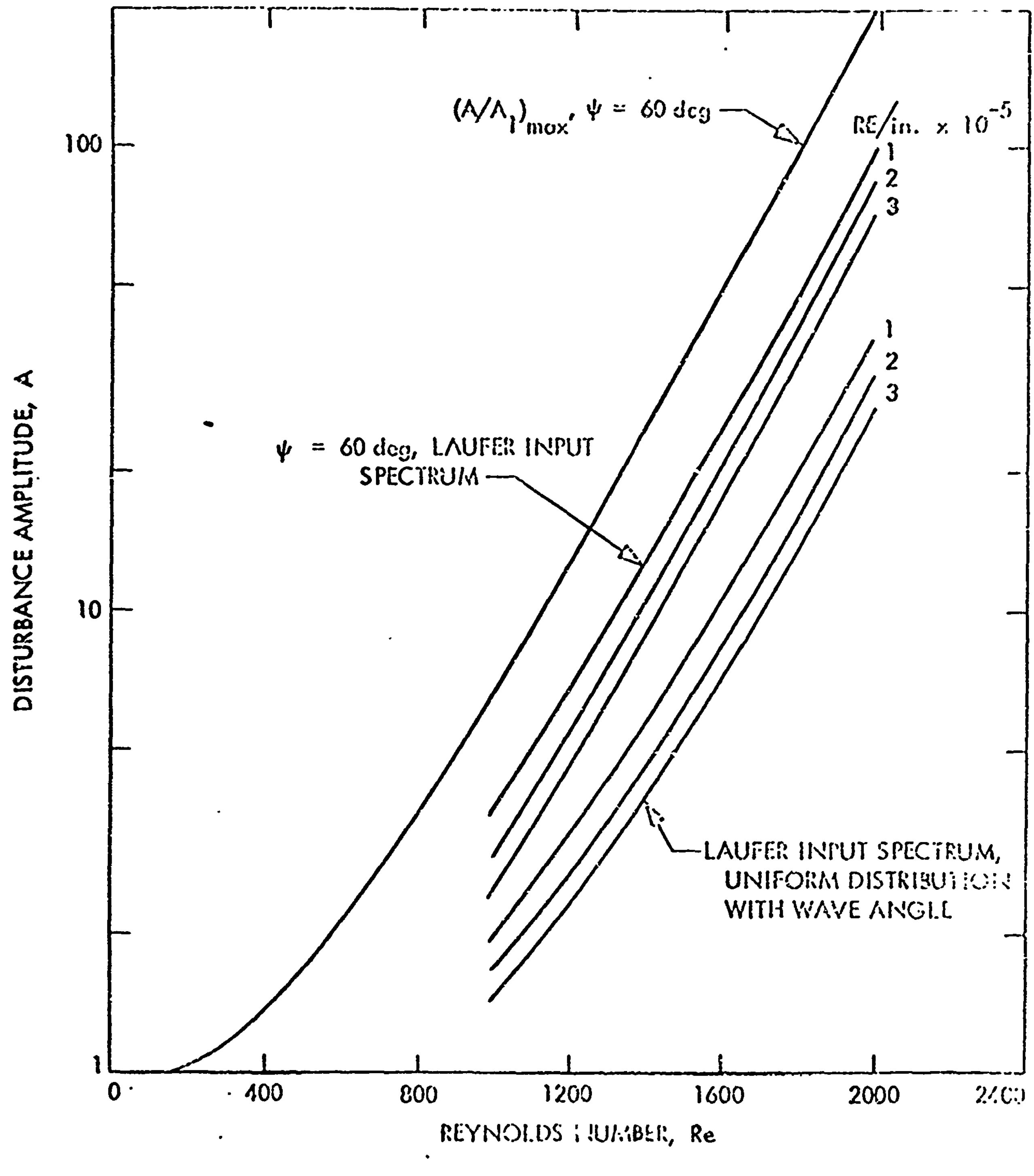

Figure 12. Effect of Various Assumptions Concerning Distribution of Input Energy with Frequency and Wave Angle on Disturbance Amplitude. $M_1 = 4.5$. Insulated Wall, $T_0^* = 311^{\circ}K$ (Mack 1969)

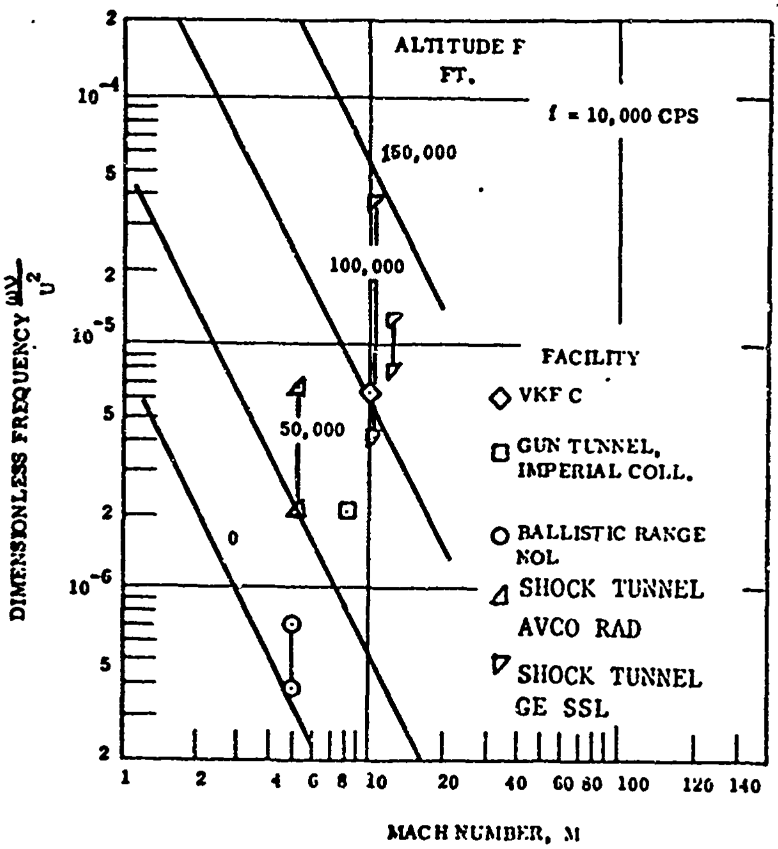

Figure 13. Dimensionless Frequency in Flight and in Various Test Facilities for a 10 kHz Disturbance (Reshotko 1968)

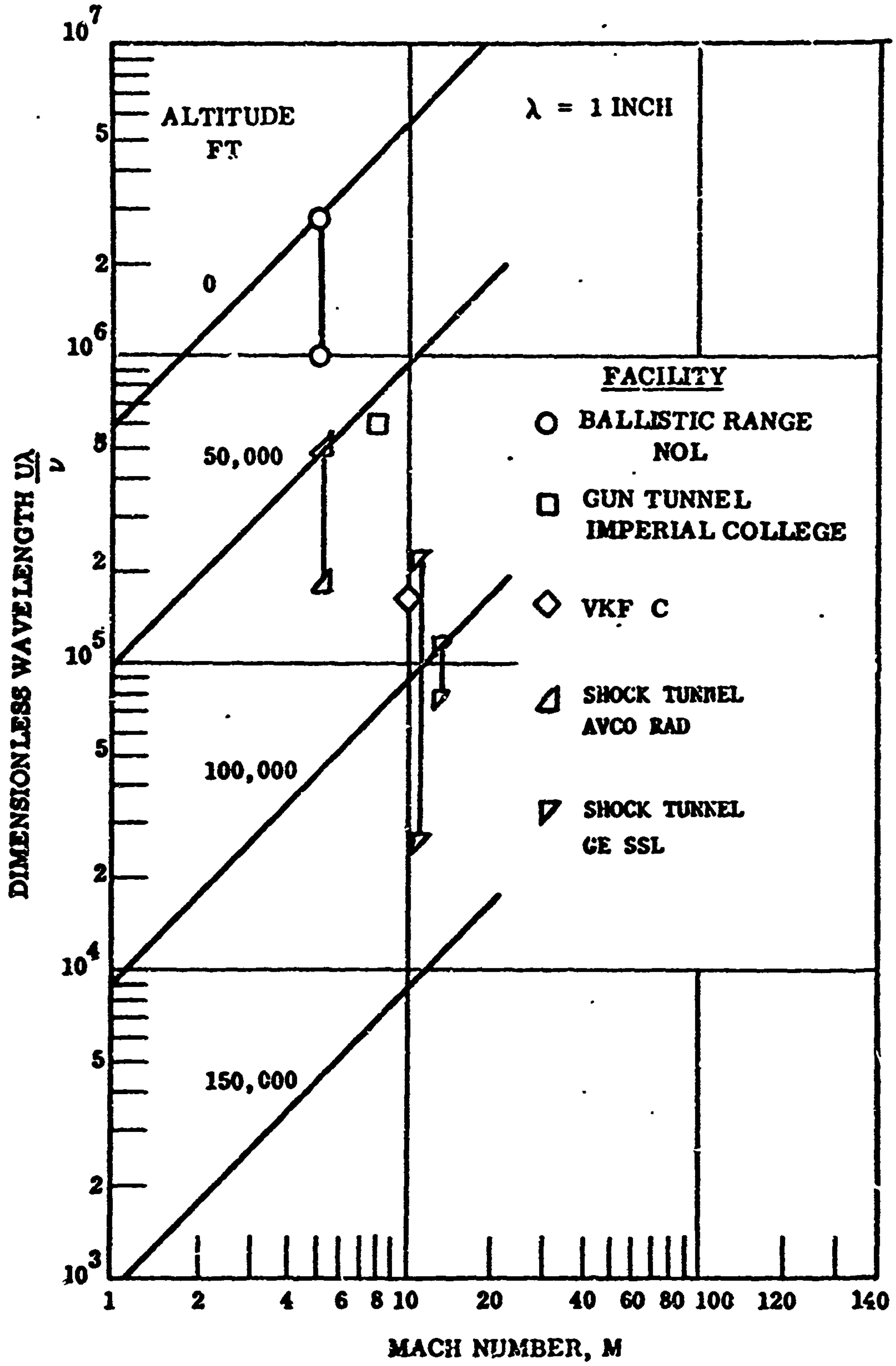

Figure 14. Dimensionless Wavelength in Flight and in Various Facilities for a Disturbance of Physical Wavelength of 1 Inch (Reshotko 1968)

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13. ABSTRACT

A review is given of boundary layer stability and transition. The normal modes procedures as they apply to boundary layers are briefly reviewed and the mechanism of instability is discussed. It is shown how normal modes results may be used to give guidance regarding the factors affecting transition. Some remarks are made about the prediction of transition and about the fixing of transition. It is concluded that the process of transition from laminar to turbulent flow remains unsolved. However, significant inroads into inroads into the understanding of transition are now possible because of our ability to do sophisticated theoretical and experimental studies of the stability of laminar boundary layers.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fluid Mechanics						
Boundary Layer Stability						
Transition						
Normal Modes						
Surface Curvature						